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Revisiting the Hanbury Brown-Twiss Setup for Phase Fluctuating Bose Gases

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The Hanbury Brown-Twiss experiment has proved to be an effective means of measuring two-point correlation function of identical particles. We analyze experimental observation of stripes formation of a phase fluctuating Bose-Einstein condensates in a highly elongated 3D traps [Dettmer *et al.*, Phys. Rev. Lett. **87**, 160406 (2001)] by means of axial two-point correlation functions. We also predict that the stripes are present in quasi-1D Bose gas in the mean-field as well as in the hard-core bosons regimes. These stripes can be realized by measuring the axial two-point correlation functions by using the Bragg interferometric method which is similar to the original Hanbury Brown and Twiss experimental setup.

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I. INTRODUCTION

Since the pioneering works on the realization of Bose-Einstein condensates (BEC) [1] of alkali-atoms, a great variety of experimental and theoretical investigations have probed the macroscopic phase coherence of confined quantum gases. For a trapped 3D BEC well below the transition temperature T_c, experiments have confirmed the macroscopic phase coherence by measuring the correlation length which is equal to the condensate size [2]. However, the phase coherence strongly depends on the shape of the confining potential which can be control at will. It was theoretically proposed that the axial phase fluctuations of an elongated 3D BEC can be very large in the equilibrium state, where the density fluctuations are strongly suppressed [3]. The axial phase coherence length in the elongated systems can be smaller than the axial size and this is referred as the quasi-condensates. Similarly, quasi-1D Bose gas in the mean-field regime [4] forms a quasi-condensate [5] with a large phase fluctuations even at temperatures as low as $0.1T_c$ [6]. The quasi-1D Bose gas behaves like a hard-core bosons (commonly known as Tonks-Girardeau Gas) [7] when the strength of the two-body potential is very strong which has been observed experimentally [8]. The hard-core bosonic systems have a very strong vacuum phase fluctuations which prevents from being a (quasi-)condensate even at $T = 0$ and mimicking the exclusion principle for fermions. The most striking features of the phase fluctuating 3D BEC is that the stripes formation in the axial density profile of the system [9].

The celebrated Hanbury Brown and Twiss (HBT) experiment [10] in which the spatial second-order correlation function $C_2(z)$ of a light source is characterized by measuring the correlations of intensity fluctuations in the wave field. The original idea of the HBT experiment is that by measuring the intensity correlations between two separated beams, they essentially compared the intensities at two different points in the unseperated beam [11]. The equal-times second-order correlation function provides information on the relative spatial distribution of pairs of identical particles. Therefore, the stripes formation can be understand by analyzing the second-order correlation function if it shows oscillatory behavior.

In this Letter, we understand the experimental observations of the stripes formation in the axial density profile of a phase fluctuating 3D BEC by means of the axial two-particle correlation functions. Next, we propose that the same stripes formation in the axial density profile can be observed in quasi-1D Bose gas in the mean-field as well as in the hard-core bosons regimes. We also discuss how to realize the stripes formation by a suitable choices of the Bragg pulse in the Bragg interferometer which is similar to the HBT experiment.

Two-point correlation function: We consider a 3D BEC confined in an elongated harmonic trap, where the repulsive mean-field interaction energy exceeds the radial ($\hbar\omega_0$) and the axial ($\hbar\omega_z$) trap energies. At $T = 0$, the density profile has the well-known form $n_0(\rho, z) = (\mu/g)(1 - \rho^2/R_0^2 - z^2/Z_0^2)$, where $\mu = 0.5\hbar\omega_z[15Na\omega_0^2/a_z\omega_z^2]^{2/5}$ is the zero-temperature chemical potential, and $g = 4\pi a\hbar^2/m$ is the two-body interaction strength. Also, $R_0 = \sqrt{2\mu/m\omega_0^2}$ and $Z_0 = \sqrt{2\mu/m\omega_z^2}$ are the radial and the axial size of the condensate, respectively. Due to the repulsive mean-field energy, density fluctuations are strongly suppressed in a trapped BEC. Therefore, the bosonic field operator describing the condensate can be written in the form $\hat{\psi}(\mathbf{r}) = \sqrt{n_0(\mathbf{r})}exp[i\hat{\phi}(\mathbf{r})]$, where the phase operator $\hat{\phi}(\mathbf{r})$ is defined as

$$\hat{\phi}(\mathbf{r}, t) = \sum_\nu \sqrt{\frac{2g}{\hbar\omega_\nu}}\psi_\nu(\mathbf{r})e^{-i\omega_\nu t}\hat{\alpha}_\nu + H.c. \tag{1}$$

Here, $\hat{\alpha}_\nu$ is the annihilation operator of the quasiparticle excitation with quantum numbers ν and energy $\hbar\omega_\nu$; ψ_ν is the corresponding quasiparticles normalized wave functions.

The normalized two-particle correlation function is de-

fined as

$$C_2[\{\mathbf{r}_i\}] = \prod_{i=1}^{4}[\sqrt{n_0(\mathbf{r}_i)}]^{-1} < \hat{\psi}^\dagger(\mathbf{r}_1)\hat{\psi}^\dagger(\mathbf{r}_2)\hat{\psi}(\mathbf{r}_3)\hat{\psi}(\mathbf{r}_4) >$$
$$= e^{-\frac{1}{2}<[\hat{\phi}(\mathbf{r}_1)+\hat{\phi}(\mathbf{r}_2)-\hat{\phi}(\mathbf{r}_3)-\hat{\phi}(\mathbf{r}_4)]^2>} = e^{-\frac{1}{2}F(\{\mathbf{r}_i\})}. \quad (2)$$

Due to the strong suppression of the density fluctuations, the normalized density correlation function of the trapped condensate is constant, *i.e.* $C_2(r_1, r_2, r_2, r_1) = 1$. Therefore, a simple measurement of the density correlations in the condensate is not enough to describe coherence properties. However, by measuring the density correlations in the interference pattern generated by two spatially displaced copies of a parent BEC, it is possible to correlate the bosonic field operator $\hat{\psi}(\mathbf{r})$ at four different positions and extract the $C_2(r_1, r_2, r_3, r_4)$ [12].

3D cigar-shaped BEC: As described in Ref. [9], the 3D condensate consists of $N = 5 \times 10^5$ atoms of ^{87}Rb. The radial and axial trapping frequencies are $\omega_0 = 2\pi \times 365$ Hz and $\omega_z = 2\pi \times 14$ Hz ($a_z = \sqrt{\hbar/m\omega_z} = 2.879\mu m$), respectively. The condensate is already elongated along the longitudinal z axis due to the aspect ratio $\lambda = \omega_0/\omega_z = 26$. In addition, further radial compression or expansion of the condensate are obtained by applying the superimposed blue detuned optical dipole trap to the magnetic trap. In the experiment [9], they performed the measurements for the radial trapping frequencies ω_0 between $2\pi \times 138$ Hz and $2\pi \times 715$ Hz corresponding to aspect ratios λ between 9.8 and 51, respectively.

The excitations of a cigar-shaped BEC can be divided into two regimes: "low-energy" axial excitations with energy $\hbar\omega_z \leq E_a << \hbar\omega_0$, and "high-energy" radial excitations with $E_r \geq \hbar\omega_0 >> \hbar\omega_z$. As pointed out in Ref. [3] that the low-energy axial excitations have wave-lengths larger than R_0 and exhibit a pronounced 1D behavior which gives the most important contribution to the low-energy axial phase fluctuations. The low-energy axial modes have the energy spectrum $\epsilon_j = \hbar\omega_z\sqrt{j(j+3)/4}$ [13]. The normalized wave functions ψ_j of these quasi-particle modes have the form

$$\psi_j(\mathbf{r}) = \sqrt{\frac{(j+2)(2j+3)}{8\pi(j+1)R_0^2 Z_0}} P_j^{(1,1)}(\tilde{z}), \quad (3)$$

where $P_j^{(1,1)}(\tilde{z})$ are Jacobi polynomials and $\tilde{z} = z/Z_0$ is a dimensionless variable.

Using Eq. (1) for the phase operator, $F(\{\mathbf{r}_i\})$ becomes,

$$F(\{z_i\}) = \sum_{j=1}^{j_{\max}} \frac{2aa_z^2}{Z_0 R_0^2} \frac{(j+2)(2j+3)}{(j+1)\sqrt{j(j+3)}} coth[\frac{\hbar\omega_j}{2k_B T}]$$
$$\times [P_j^{(1,1)}(\tilde{z}_1) + P_j^{(1,1)}(\tilde{z}_2)$$
$$- (P_j^{(1,1)}(\tilde{z}_3) + P_j^{(1,1)}(\tilde{z}_4))]^2. \quad (4)$$

The upper cut-off limit ($j_{\max}$) on the summation can be obtained from the constraint: $\epsilon_j < \mu$. It is useful to define the space variables in the following way: $\tilde{z}_1 = (s+d)/2 = -\tilde{z}_2$, and $\tilde{z}_3 = (s-d)/2 = -\tilde{z}_4$. Here, d is the displacement between the two interfering condensate copies and s is the relative distance between the positions in the interference pattern at which the particle densities are evaluated. This new choice of variables has been realized in the experiment [12].

In Fig.1 and Fig.2, we plot the axial two-particle correlation function vs the relative separation at $T = 0$ and $T = 0.6T_c$, respectively, for three different choices of the aspect ratio.

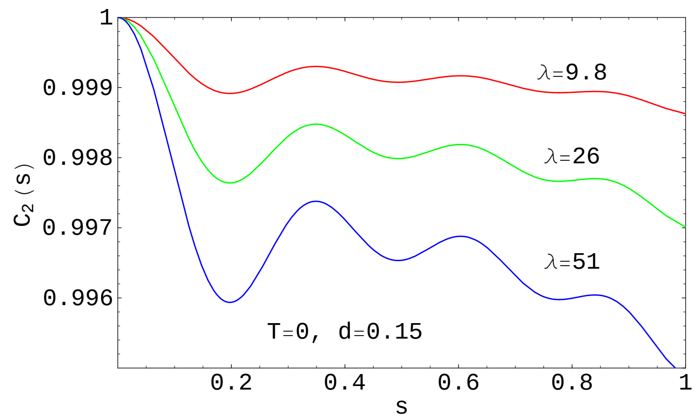

FIG. 1. (Color online) Plots of the axial two-particle correlation functions vs the relative separation.

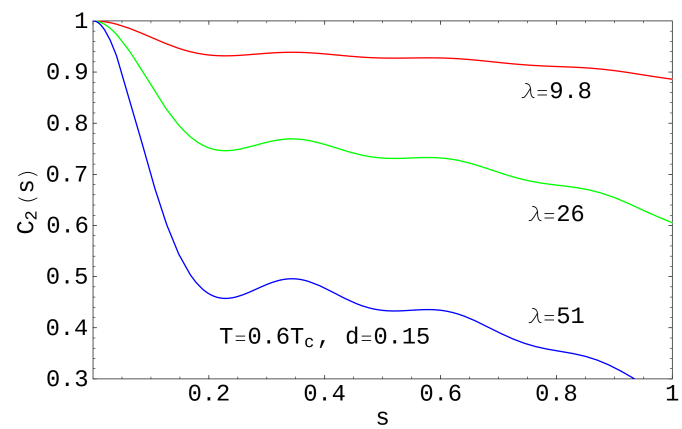

FIG. 2. (Color online) Plots of the axial two-particle correlation functions vs the relative separation.

The axial two-particle correlation function at zero temperature has oscillatory behavior with very small amplitude. It shows little evidence of the stripes formation even at zero temperature. Nevertheless, we do not claim that stripes are present even at $T = 0$ because it is almost one for a large separation. The minima of the $C_2(s)$ at various relative separations are decreasing with the increasing of the aspect ratio λ. It implies that vacuum phase fluctuation increases with the aspect ratio.

Fig.2 shows the oscillatory behavior with large amplitudes compared to the zero temperature case. These prominent deep valleys implies the presence of stripes in the axial density distribution due to the strong thermally excited phase fluctuations and the large aspect ratio λ. The minima of the $C_2(d, s)$ at various relative separations are decreasing with the increasing of the temperature and the aspect ratio λ.

In the actual experiment [9], the stripes formation is observed after the ballistic expansion of the atomic cloud. However, it does not rule out the possibility of the stripes formation in static BEC *i.e.*, without ballistic expansion. According to the Ref. [9], the observation of the stripes formation after the ballistic expansion of the cloud is due to the rapidly decreasing of the mean-field interaction and the axial velocity fields are then converted into the density distribution. In our studies we find that the stripes formation in the axial density profile is already present even before switching off the trap. Therefore, the stripes formation are not necessarily due to the axial velocity fields which are converted into density modulations during the ballistic expansion, as stated in Ref. [9]. The width of the stripes are too small to be observed directly in-trap due to the lack of the experimental resolution, but it is seen after expansion of the cloud since the stripes in the static cloud is enlarged during the ballistic expansion of the cloud.

Quasi-1D Bose gas in the mean-field regime: Now we consider quasi-1D Bose gas in the mean-field regime as described in Ref. [4]. In particular, the system consists of $N \sim 10^4$ atoms of ^{23} Na in the trap with axial trapping frequency $\omega_z = 2\pi \times 3.5$ Hz, and radial trapping frequency $\omega_0 = 2\pi \times 360$ Hz. The low-energy excitation spectrum is given by $\omega_j = \omega_z\sqrt{j(j+1)/2}$ and the corresponding normalized eigenfunctions are $\psi_j(z) = \sqrt{\frac{2j+1}{2Z_0}}P_j(z/Z_0)$, [14] where $P_j(z/Z_0)$ is the Legendre polynomial in z and $Z_0 = a_z(2.998Naa_z/a_0^2)^{1/3}$ is the Thomas-Fermi half length.

Using Eq. (1) for the phase operator, $F(\{\mathbf{r}_i\})$ for quasi-1D system becomes,

$$F(\{z_i\}) = \sum_{j=1}^{j_{\max}} \frac{aa_z}{a_0^2}\frac{a_z}{Z_0}\frac{(2j+1)}{\sqrt{2j(j+1)}} coth[\frac{\hbar\omega_j}{2k_BT}] \times [P_j(\tilde{z}_1) + P_j(\tilde{z}_2) - P_j(\tilde{z}_3) - P_j(\tilde{z}_4)]^2. \quad (5)$$

Using the expression for $F(\{z_i\})$, we plot two-point correlation functions vs the relative distance at $T = 0$ and $T = 0.6T_c$ in Fig.3 and Fig.4, respectively.

Similar to the 3D BEC in very elongated traps, the vacuum phase fluctuations are not able to produce the stripes in quasi-1D Bose gas in the mean-field regime, However, at large temperature, the two-point correlation functions have prominent oscillatory behavior with large amplitudes, compared to the zero temperature case. Therefore, the stripes are present in the quasi-1D Bose gas in the mean-field regime at large temperature.

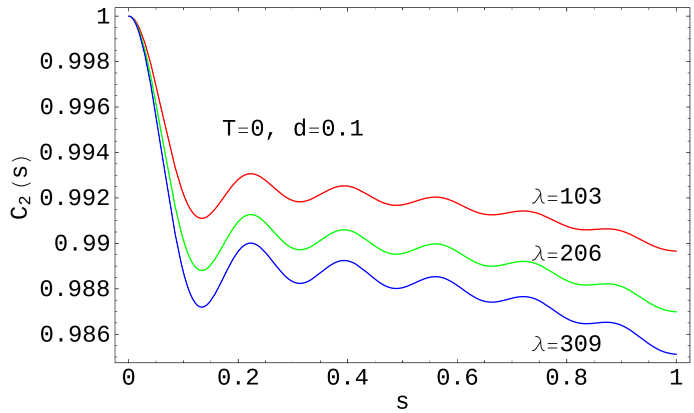

FIG. 3. (Color online) Plots of two-point correlation functions vs the relative distance for various trapping aspect ratio.

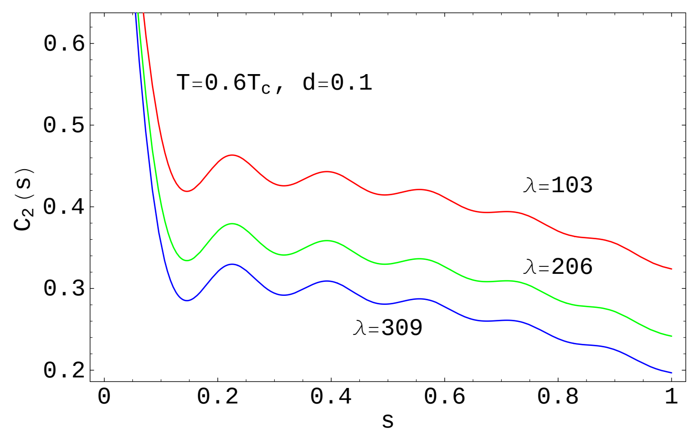

FIG. 4. (Color online) Plots of two-point correlation functions vs the relative distance for various trapping aspect ratio.

Quasi-1D Bose gas in the hard-core bosons regime: Let us consider quasi-1D Bose gas in the hard-core bosons regime. The low-energy excitation spectrum is given by $\epsilon_j = j\hbar\omega_z$ and the corresponding normalized eigenfunctions are $\psi_j(z) = \sqrt{\frac{2}{\pi Z_0}}T_j(z/Z_0)$, where $T_j(z/Z_0)$ is the first-order Chebyshev polynomial in z. Here, $Z_0 = \sqrt{2N}a_z$ is the Thomas-Fermi half-length of the hard-core bosons system with the chemical potential $\mu = N\hbar\omega_z$ [15].

Using Eq. (1) for the phase operator, $F(\{\mathbf{r_i}\})$ for quasi-1D system in the hard-core bosons regime at $T = 0$ becomes

$$F[z_i] = \sum_{j=1}^{N} \frac{1}{j}[T_j(\tilde{z}_1) + T_j(\tilde{z}_2) - T_j(\tilde{z}_3) - T_j(\tilde{z}_4)]^2. \quad (6)$$

Fig. 5 shows the two-point correlation function of the quasi-1D Bose gas in the hard-core regime at $T = 0$.

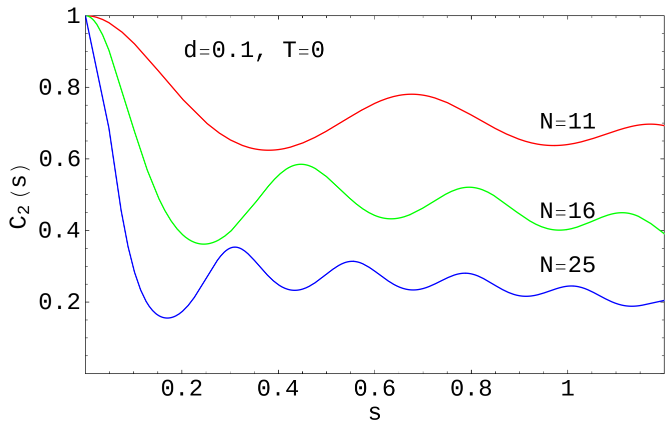

FIG. 5. (Color online) Plots of two-point correlation functions vs the relative distance for various number of hard-core atoms.

Note that Fig.5 is valid only when $s > 1/2N$ due to the hydrodynamic approximation. This hydrodynamic approximation is failed to describe the short-range correlations due to the strong interactions when $s < 1/2N$. Fig. 5 shows that the two-point correlation functions have the oscillatory behavior with large amplitude even at zero temperature. This implies that stripes are already present in the hard-core bosons regime even at zero temperature due to the strong quantum phase fluctuations. Note that the amplitude of the oscillations in the zero temperature two-point correlation functions is very very small in the cigar-shaped 3D BEC as well as in the quasi-1D Bose gas in the mean field regime, compared to the quasi-1D Bose gas in the hard-core regime. The two-point correlation functions with large amplitude of the oscillations implies the fermionic like behavior in the hard-core Bose gas at zero temperature.

Detection: The realization of the stripes formation is possible by measuring the axial two-particle correlation function. The Bragg interferometric method presented in Ref. [12] to measure the two-particle correlation function is analogous to the original HBT experiment [10]. Therefore, the prominent valleys in the axial two-particle correlation function should be observe by using the Bragg interferometric method as described in Ref. [12] with a proper choices of the wave vector k and the time interval (Δt) between the two Bragg pulses such that the distances ($d = 2\hbar k \Delta t/m$) between two auto correlated copies should be $d \sim 0.1 - 0.2$. We have checked that for other choices of d, $d \geq 0.2$, the oscillatory behavior in the two-point correlation functions washes out. This is due to the fact that the two interfering matter waves are not HBT-correlated when $d \geq 0.2$. Here we mean HBT-correlated in a sense that the matter waves will produce an HBT effect at the output ports, as opposed to the different question of the correlations in the parent condensate [11]. In fact, the two matter waves are HBT-correlated only if they are interfering within a coherence time (τ), the characteristic time scale in the experiment. The relative distance d between two interfering BECs at the output ports of the interferometer can be controlled by varying the time interval (Δt). When Δt is small such that $d \sim 0.1 - 0.2$, we expect the correlations to be maximal and shows the HBT effect. For large $\Delta t > \tau$, the distance between two interfering BECs is large so that it becomes HBT-uncorrelated and then it does not produce valleys in the interference patterns. The time interval, Δt, before colliding two copies should be less than the coherence time τ which is inversely proportional to the temperature of the condensate *i.e.* $\Delta t < \tau \sim \hbar/k_B T$, where T is the temperature of the parent condensate [11].

In conclusion, we have understand the stripes in the axial density profile of a strong phase fluctuating 3D cigar-shaped BECs in terms of the two-point correlation function. We have also predicted that the stripes are present in the quasi-1D Bose gas in the mean-field regime at large temperature as well as in the 1D hard-core bosonic systems even at zero temperature due to to the strong quantum phase fluctuations. We have also pointed out that one can realize the stripes by a proper choices of the parameters in the Bragg interferometer experiment which is similar to the HBT setup. This experiment would provide the direct realization of the stripes formation, which is of fundamental importance in the phase fluctuating BECs.

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